- **Alternating Current:**
  The current whose magnitude changes with time and direction reverses periodically is called alternating current. a) Alternating emf E and current I at any time am given by:
  $$E = E_0 \sin \omega t$$
  Where $E_0 = NBA\omega$
  $$I = I_0 \sin(\omega t - \phi)$$
  Where $I_0 = \frac{NBA\omega}{R}$
  $$\omega = 2\pi n = \frac{2\pi}{T}$$
  Where T is the time period.
- **Values of Alternating Current and Voltage**
  a) Instantaneous value:
  It is the value of alternating current and voltage at an instant t.
  b) Peak value:
  Maximum values of voltage E_0 and current I_0 in a cycle are called peak values.
  c) Mean value:
  For complete cycle,
  $$< E >= \frac{1}{T}\int_0^T Edt = 0$$
  $$< I >= \frac{1}{T}\int_0^T Idt = 0$$
  Mean value for half cycle: $E_{mean} = \frac{2E_0}{\pi}$
  d) Root – mean- square (rms) value:
  $$E_{rms} = (< E^2 >)^{1/2} = \frac{E_0}{\sqrt{2}} = 0.707E_0 = 70.7\%E_0$$
  $$I_{rms} = (< I^2 >)^{1/2} = \frac{I_0}{\sqrt{2}} = 0.707I_0 = 70.7\%I_0$$
  RMS values are also called apparent or effective values.
- **Phase difference Between the EMF (Voltage) and the Current in an AC Circuit**
  a) For pure resistance:
  The voltage and the current are in same phase i.e. phase difference $\phi = 0$
  b) For pure inductance:

The voltage is ahead of current by $\frac{\pi}{2}$ i.e. phase difference $\phi = +\frac{\pi}{2}$.

c) For pure capacitance:

The voltage lags behind the current by $\frac{\pi}{2}$ i.e. phase difference $\phi = -\frac{\pi}{2}$

- **Reactance:**

a) Reactance

$$X = \frac{E}{I} = \frac{E_0}{I_0} = \frac{E_{rms}}{I_{rms}} \pm \pi / 2$$

b) Inductive reactance

$X_L = \omega L = 2\pi n L$

c) Capacitive reactance

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi n C}$$

- **Impedance:**

Impedance is defined as,

$$Z = \frac{E}{I} = \frac{E_0}{I_0} = \frac{E_{rms}}{I_{rms}} \phi$$

Where ϕ is the phase difference of the voltage E relative to the current I.

a) For L – R series circuit:

$$Z_{RL} = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega L^2}$$

$$\tan\phi = \left(\frac{\omega L}{R}\right) or \phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

b) For R – C series circuit:

$$Z_{RC} = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$\tan\phi = \frac{1}{\omega CR} \text{ Or } \phi = \tan^{-1}\left(\frac{1}{\omega CR}\right)$$

c) For L – C series circuit:

$$Z_{LCR} = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\tan\phi = \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R} \text{ Or } \phi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

- **Conductance:**

Reciprocal of resistance is called conductance.

$$G = \frac{1}{R} mho$$

- **Power in and AC Circuit:**

a) Electric power = (current in circuit) x (voltage in circuit)

P = IE

b) Instantaneous power:

$P_{inst} = E_{inst} \times I_{inst}$

c) Average power:

$$P_{av} = \frac{1}{2} E_0 I_0 \cos\phi = E_{rms} I_{rms} \cos\phi$$

d) Virtual power (apparent power):

$$= \frac{1}{2} E_0 I_0 = E_{rms} I_{rms}$$

- **Power Factor:**

a) Power factor

$$\cos\phi = \frac{P_{av}}{P_v} = \frac{R}{Z}$$

b) For pure inductance

Power factor, $\cos\phi$=1

c) For pure capacitance

Power factor, $\cos\phi$=0

d) For LCR circuit

Power factor, $\cos\phi = \dfrac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$

$$X = \left(\omega L - \frac{1}{\omega C}\right)$$

- **Wattless Current:**

The component of current differing in phase by $\frac{\pi}{2}$ relative to the voltage, is called wattles current.

- **The rms value of wattless current:**

$$= \frac{I_0}{\sqrt{2}} \sin\phi$$

$$= I_{rms} \sin\phi = \frac{I_0}{\sqrt{2}} \left(\frac{X}{Z}\right)$$

- **Choke Coil:**
  a) An inductive coil used for controlling alternating current whose self- inductance is high and resistance in negligible, is called choke coil.
  b) The power factor of this coil is approximately zero.
- **Series Resonant Circuit**
  a) When the inductive reactance (XL) becomes equal to the capacitive reactance (XC) in the circuit, the total impedance becomes purely resistive (Z=R).
  b) In this state, the voltage and current are in same phase (ϕ = 0), the current and power are maximum and impedance is minimum. This state is called resonance.
  c) At resonance,

$$\omega_r L = \frac{1}{\omega_r C}$$

  Hence, resonance frequency is,

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

  d) In resonance, the power factor of the circuit is one.
- **Half – Power Frequencies:**
  Those frequencies f_1 and f_2 at which the power is half of the maximum power (power at resonance), i.e., f_1 and f_2 are called half – power frequencies.

$$P = \frac{1}{2}P_{\max}$$

$$I = \frac{I_{\max}}{\sqrt{2}}$$

$$\therefore P = \frac{P_{\max}}{2}$$

- **Band – Width:**
  a) The frequency interval between half – power frequencies is called band – width.

$$\therefore \text{ Bandwidth } \Delta \text{f} = f_2 - f_1$$

  b) For a series LCR resonant circuit,

$$\Delta \text{f} = \frac{1}{2\pi}\frac{R}{L}$$

- **Quality Factor (Q):**

$$Q = 2\pi \times \frac{\text{Maximum energy stored}}{\text{Energy dissipated per cycle}}$$

$$= \frac{2\pi}{T} \times \frac{\text{Maximum energy stored}}{\text{Mean power dissipated}}$$

Or

$$Q = \frac{\omega_r L}{R} = \frac{1}{\omega_r CR} = \frac{f_r}{(f_2 - f_1)} = \frac{f_r}{\Delta f}$$