# Chapter-10

# Vector Algebra

- Position vector of a point P (x, y) is given as $\overline{\quad} = \bar{r}) = x\hat{i} + y\hat{j} + z\hat{k}$ and its magnitude by $\sqrt{x^2 + y^2 + z^2}$
- The scalar components of a vector are its direction ratios, and represent its projections along the respective axes.
- The magnitude (r), direction ratios (a, b, c) and direction cosines (l, m, n) of any vector are related as: $l = \frac{a}{r}, m = \frac{b}{r}, n = \frac{c}{r}$
- The vector sum of the three sides of a triangle taken in order is $\overline{O}$
- The vector sum of two conidial vectors is given by the diagonal of the parallelogram whose adjacent sides are the given vectors.
- The multiplication of a given vector by a scalar λ, changes the magnitude of the vector by the multiple |λ|, and keeps the direction same (or makes it opposite) according as the value of λ is positive (or negative).
- For a given vector $\bar{a}$, the vector $\hat{a} = \frac{\bar{a}}{|\bar{a}|}$ gives the unit vector in the direction of $\bar{a}$
- The position vector of a point R dividing a line segment joining the points P and Q whose position vectors are $\bar{a}$ *and* $\bar{b}$ respectively, in the ratio $m:n$

(i) internally, is given by $\frac{\bar{\ } + \bar{\ }}{m+n}$

(ii) externally, is given by $\frac{\bar{\ } - \bar{\ }}{m-n}$

- The scalar product of two given vectors $\bar{a}$ and $\bar{b}$ having angle θ between them is defined as

$\bar{\ }\ \bar{\ } = |\bar{a}||\bar{b}|\cos\theta$

Also, when $\bar{a}\,.\,\bar{b}$ is given, the angle $\theta$' between the vectors $\bar{a}$ *and* $\bar{b}$ may be determined by

$$cos\theta = \frac{\bar{\ }\ \ \bar{\ }}{|\bar{a}||\bar{b}|}$$

- If $\theta$ is the angle between two vector $\bar{a}$ and $\bar{b}$ , then their cross product is given as

  $\bar{a} \times \bar{b} = |\bar{a}||\bar{b}|\sin\theta\hat{n}$ where $\hat{n}$ is a unit vector perpendicular to the plane containing $\bar{a}$ *and* $\bar{b}$.

  Such that $\bar{a}$ , $\bar{b}$, $\hat{n}$ form right handed system of coordinate axes.

- If we have two vectors $\bar{a}$ *and* $\bar{b}$ given in component form as $\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\lambda$ any scalar, then, $a + b = (a_1 + b_1)\hat{i} + (a_2 + b_2)\hat{j} + (a_3 + b_3)\hat{k}$

  $\lambda\bar{a} = (\lambda a_1)\hat{i} + (\lambda a_2)\hat{j} + (\lambda a_3)\hat{k};$

  $\bar{a}.\bar{b} = a_1b_1 + a_2b_2 + a_3b_3$

  and $\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$