Chapter-3

Matrices

- A matrix is an ordered rectangular array of numbers or functions.
- A matrix having m rows and n columns is called a matrix of order m × n.
- $[a_{ij}]_{m\times 1}$ is a column matrix.
- $[a_{ij}]_{1\times n}$ is a row matrix.
- An m × n matrix is a square matrix if m = n.
- $A = A = [a_{ij}]_{m\times n}$ is a diagonal matrix if $a_{ij} = 0$, when $i \neq j$
- $A = \left[a_{ji}\right]_{n\times n}$ is a scalar matrix if $a_{ij} = 0$ when $i \neq j$, $a_{ij} = k$ (k is *some constant), when I=j.*
- $A = \left[a_{ij}\right]_{n\times n}$ is an identity matrix, if $\cdot\ {}_{ij} = 1$, *when* $i = j$, $a_{ij} = 0$, *when* $i \neq j$.
- A zero matrix has all its elements as zero.
- $A = \left[a_{ij}\right] = \left[b_{ij}\right] = B$ if (i) A and B are of same order, (ii) for all possible values of i *and* j.
- $kA = k\left[a_{ij}\right]_{m\times n} = \left[k\left(a_{ij}\right)\right]_{m\times n}$
- – A = (–1)A
- A – B = A + (–1) B
- A + B = B + A
- (A + B) + C = A + (B + C), where A, B and C are of same order.
- k(A + B) = kA + kB, where A and B are of same order, k is constant.
- (k + l) A = kA + lA, where k and l are constant.
- *If* $A = [a_{ij}]_{m\times n}$ *and* $B = [b_{jk}]_{n\times p}$, *then* $AB = C = [c_{ik}]_{m\times p}$, where $C_{tl} = \sum_{j=i}^{n} a_{ij} b_{jk}$

(i) A(BC) = (AB)C,

(ii) A(B + C) = AB + AC,

(iii) (A + B)C = AC + BC

- *If* $A = \left[a_{\mathrm{ij}}\right]_{\mathrm{m\times n}}$, *then* A' *or* $A^{\mathrm{T}} = \left[a_{\mathrm{ji}}\right]_{\mathrm{n\times m}}$
- (i) (A′)′ = A,
- (ii) (kA)′ = kA′,
- (iii) (A + B)′ = A′ + B′,
- (iv) (AB)′ = B′A′
- A is a symmetric matrix if A′ = A.
- A is a skew symmetric matrix if A′ = –A.
- Any square matrix can be represented as the sum of a symmetric and a skew symmetric matrix.
- Elementary operations of a matrix are as follows:
- (i) $\mathrm{R_1 \leftrightarrow R_j}$ or $\mathrm{C_l \leftrightarrow C_j}$

(i) $\mathrm{R_1 \rightarrow kR_i}$ or $\mathrm{C_l \leftrightarrow kC_l}$

(i) $\mathrm{R_1 \leftrightarrow R_j + kR_j}$ or $\mathrm{C_l + kC_j}$

- If A and B are two square matrices such that AB = BA = I, then B is the inverse matrix of A and is denoted by A^{-1} and A is the inverse of B.
- Inverse of a square matrix, if it exists, is unique.